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Journal of Sound and Vibration 317 (2008) 449–455
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Rapid Communication

Chaos synchronization of double Duffing systems with parameters excited by a chaotic signal

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Received 8 May 2008; accepted 8 May 2008
Handling Editor: L.G. Tham
Available online 20 June 2008

Abstract

Chaos synchronization by driving parameter for two uncoupled identical chaotic double Duffing systems is presented. Replacing two corresponding parameters of the identical systems by the same function of chaotic state variables of a third chaotic system, the synchronization or anti-synchronization of two uncoupled systems can be obtained. Numerical simulations are illustrated for either synchronization or anti-synchronization of which the occurrence depends significantly on initial conditions and on driving strength. Alternative complete synchronization and anti-synchronization is also discovered.

1. Introduction

Various synchronization phenomena are being reported for chaotic systems, such as complete synchronization (CS), anti-synchronization (AS), phase synchronization (PS), lag synchronization, and generalized synchronization [1–20,29–38]. However, most of synchronizations can only realize under the condition that there exists coupling between two chaotic systems. In practice, such as in physical and electrical systems, sometimes, it is difficult even impossible to couple two chaotic systems. In comparison with coupled chaotic systems, synchronization between the uncoupled chaotic systems has many advantages [20–29].

In this paper, synchronization of two double Duffing systems whose corresponding parameter is driven by a chaotic signal of a third system is analyzed. The chaos synchronizations of two uncoupled double Duffing systems are obtained by replacing their corresponding parameters by the same function of chaotic state variables of a third chaotic system. It is noted that whether CS or AS appear depends on the initial conditions. Besides, CS and AS are also characterized by great sensitivity to initial conditions and on the strengths of the substituted variable. It is found that CS or AS alternatively occurs under certain conditions [38–42].

This paper is organized as follows. In Section 2, a brief description of synchronization scheme based on the substitution of the strengths of the mutual coupling term of two identical chaotic double Duffing systems by

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0022-460X/$ - see front matter © 2008 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2008.05.019

the variable of a third double Duffing system is presented. In Section 3, numerical simulations are given for illustration. It is found that one can obtain CS or AS by adjusting the driving strength and initial conditions. Finally, in Section 4 conclusions are drawn.

2. Synchronization of two double Duffing systems

The famous Duffing system is

$$\ddot{x} + a\dot{x} + bx + cx^3 = d\cos\omega t \tag{1}$$

where a, b are constant parameters, $d\cos\omega t$ is an external excitation. It can be written as two first-order differential equations:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -ay - bx - cx^3 + d\cos\omega t \end{cases} \tag{2}$$

Consider the following double Duffing system:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -ay - bx - cx^3 + du \\ \dfrac{du}{dt} = v \\ \dfrac{dv}{dt} = -ev - gu - hu^3 + kx \end{cases} \tag{3}$$

It consists of two Duffing systems in which two external excitations are replaced by two coupling terms. It is an autonomous system with four states where a, b, c, d, e, g, h, and k are constant parameters of the systems. Two identical double Duffing systems to be synchronized are

$$\begin{cases} \dfrac{dx_1}{dt} = y_1 \\ \dfrac{dy_1}{dt} = -ay_1 - bx_1 - cx_1^3 + d_1u_1 \\ \dfrac{du_1}{dt} = v_1 \\ \dfrac{dv_1}{dt} = -ev_1 - gu_1 - hu_1^3 + kx_1 \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{dx_2}{dt} = y_2 \\ \dfrac{dy_2}{dt} = -ay_2 - bx_2 - cx_2^3 + d_2u_2 \\ \dfrac{du_2}{dt} = v_2 \\ \dfrac{dv_2}{dt} = -ev_2 - gu_2 - hu_2^3 + kx_2 \end{cases} \tag{5}$$

where a, b, c, d, e, g, h, and k are positive scalars, and $d_1 = d_2$ are the control inputs to be designed. The third system is also a double Duffing system:

$$\begin{cases} \dfrac{dx_3}{dt} = y_3 \\ \dfrac{dy_3}{dt} = -ay_3 - bx_3 - cx_3^3 + du_3 \\ \dfrac{du_3}{dt} = v_3 \\ \dfrac{dv_3}{dt} = -ev_3 - gu_3 - hu_3^3 + kx_3 \end{cases} \quad (6)$$

In order to obtain the chaos synchronization of systems (4) and (5), the corresponding parameters $d_1 = d_2$ of two systems are replaced by a chaotic signal $px_3 + qy_3$ of the third system (6), where p, q are constant driving strengths. The error state variables are defined:

$$\begin{cases} e_1 = x_1 - x_2 \\ e_2 = y_1 - y_2 \\ e_3 = u_1 - u_2 \\ e_4 = v_1 - v_2 \end{cases} \quad (7)$$

Giving suitable values for p, q and initial conditions, the synchronization or anti-synchronization of systems (4) and (5) can be obtained.

3. Numerical simulations

Matlab method is used to all of our simulations with time step 0.01. The parameters of two systems (4) and (5) are given as $a = 0.5$, $b = 1$, $c = 3$, $d = -2$, $e = 5$, $g = 1$, $h = 2$, $k = 2$ to ensure the chaotic behavior. To verify CS and AS, it is convenient to introduce the following coordinate transformation: $E_1 = (x_1 + x_2)$ and $e_1 = (x_1 - x_2)$ and the same transformation for y, u, and v variables. Therefore, the new coordinate systems (E_1, E_2, E_3, E_4) and (e_1, e_2, e_3, e_4) represent the sum and difference motions of the original coordinate system,

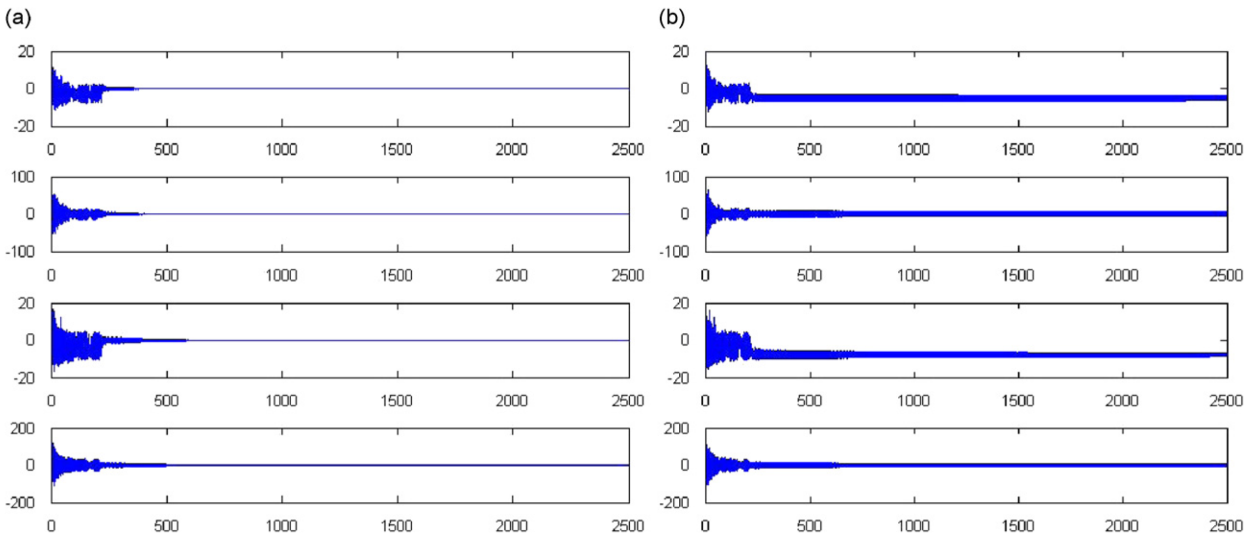

Fig. 1. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-8, -9, 0, 5)$, and $p = 10$, $q = 8$. (a) e_1, e_2, e_3, e_4 and (b) E_1, E_2, E_3, E_4.

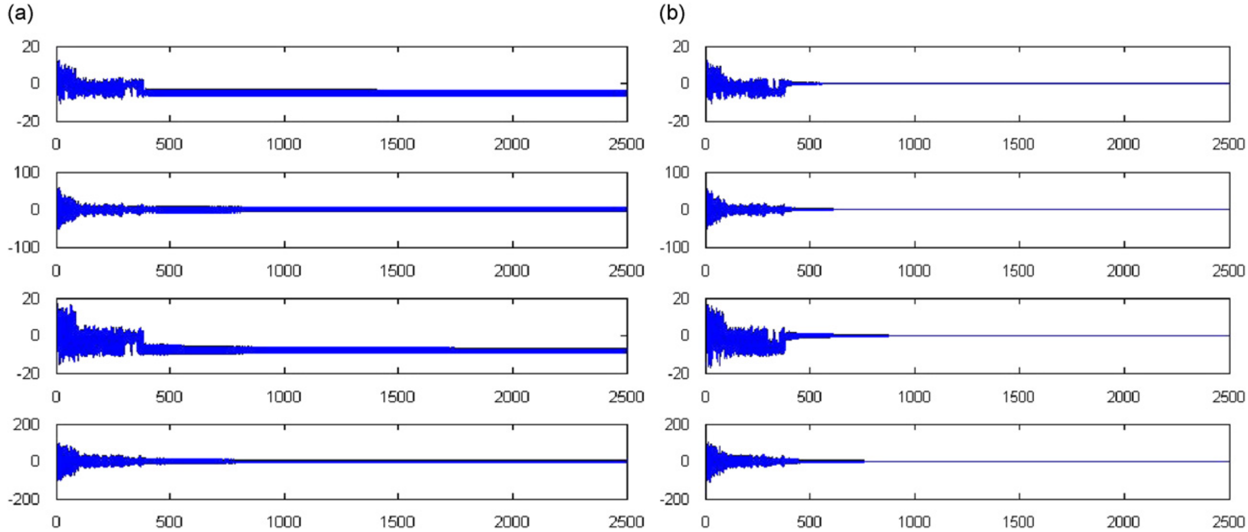

Fig. 2. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-8, -9, 0, 5)$, and $p = 10$, $q = 10$. (a) e_1, e_2, e_3, e_4 and (b) E_1, E_2, E_3, E_4.

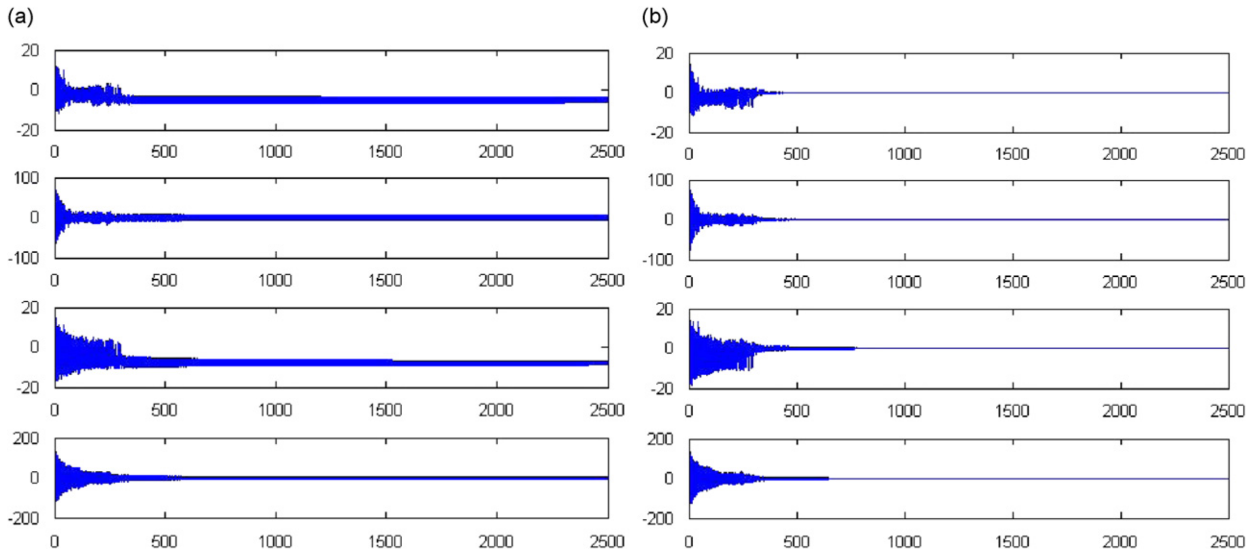

Fig. 3. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (9, 5, -7, 9)$, and $p = 10$, $q = 10$. (a) e_1, e_2, e_3, e_4 and (b) E_1, E_2, E_3, E_4.

respectively. We can easily see that (e_1, e_2, e_3, e_4) subspace represents the CS case, and (E_1, E_2, E_3, E_4) subspace the AS one.

How the synchronization phenomena depend on the initial conditions will be studied. At the beginning, we choose $(x_1, y_1, u_1, v_1) = (2, 5, 1, 0.3)$ and $(x_2, y_2, u_2, v_2) = (-8, -9, 0, 5)$ as the initial conditions of systems (4) and (5). Let the driving strengths be $p = 10$, $q = 8$ and $p = 10$, $q = 10$. Figs. 1 and 2 show the time-series of AS and CS phenomena for different driving strengths, respectively. The simulation results are shown in Fig. 1 for case (a) and in Fig. 2 for case (b). These simulation results indicate that the final state develops to CS or AS depending sensitively on driving strength in spite of the identical initial conditions in both cases. For AS case (Figs. 1(a) and (b)), the sums of the variables converge to zero, while the differences remain chaotic. For CS case (Figs. 2(a) and (b)), on the other hand, e_1, e_2, e_3, and e_4 converge to zero, while E_1, E_2, E_3, and E_4 remain chaotic.

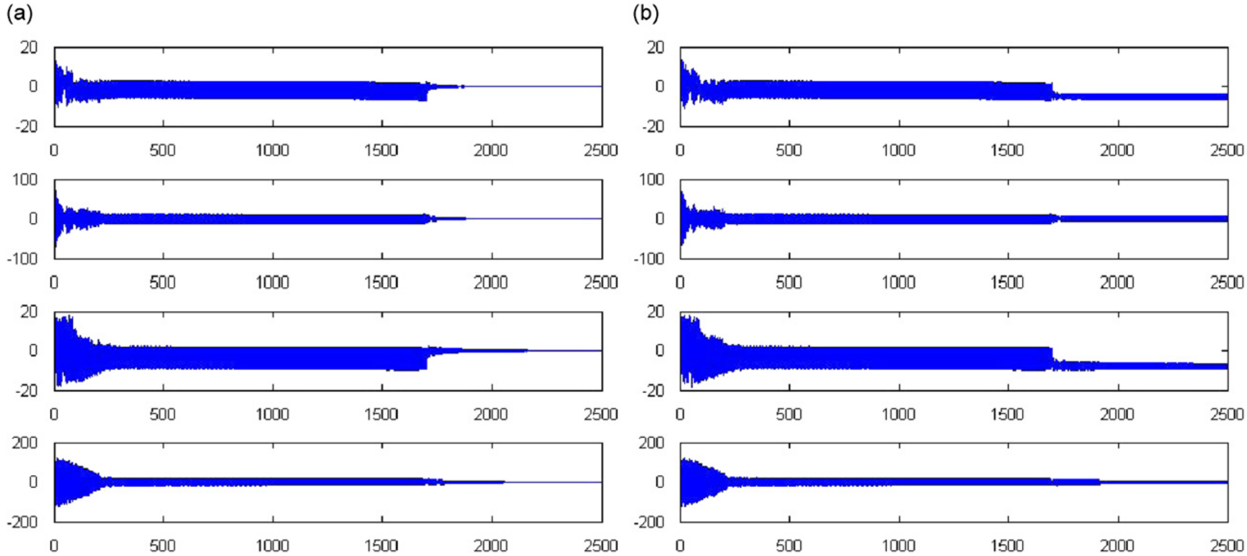

Fig. 4. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-8, -9, 0, 5)$, and $p = 10$, $q = 13$. (a) e_1, e_2, e_3, e_4 and (b) E_1, E_2, E_3, E_4.

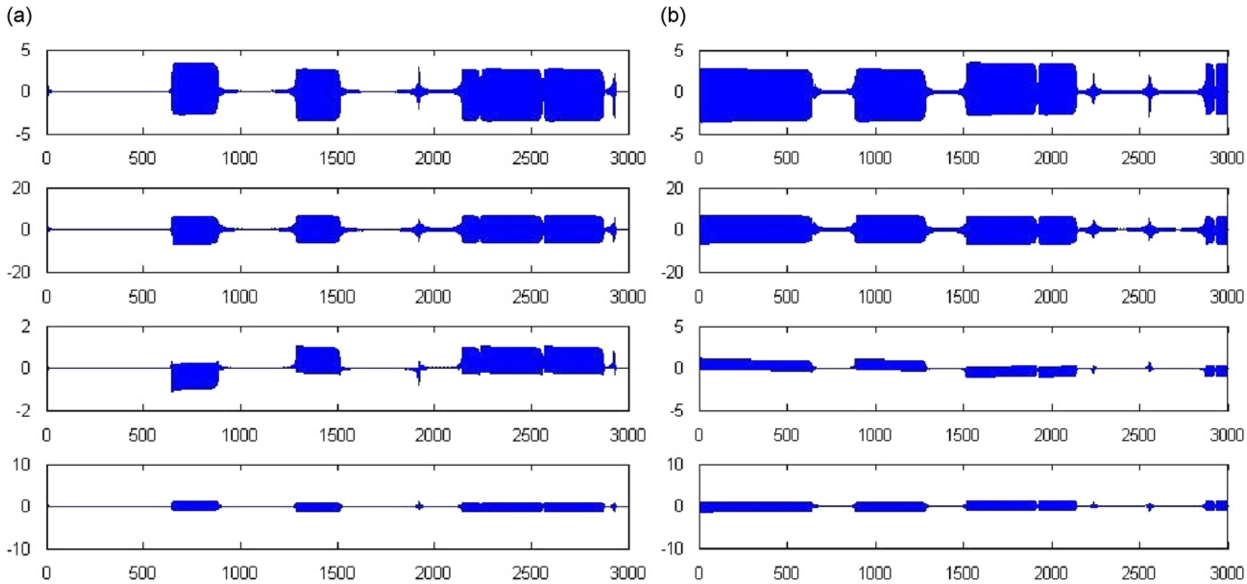

Fig. 5. Alternative CS and AS for initial condition $(x_1, y_1, u_1, v_1) = (2, 5, 1, 0.3)$, $(x_2, y_2, u_2, v_2) = (-3, 5, 2, 9)$, and $p = 12$, $q = 12$. (a) e_1, e_2, e_3, e_4 and (b) E_1, E_2, E_3, E_4.

In order to know how this phenomenon depends upon the initial conditions, different initial conditions are given for fixed driving strength. The results are shown in Figs. 3 and 4. Fig. 3(b) shows that E_1, E_2, E_3, and E_4 tend to zero. As shown in Fig. 3(a), while the e_1, e_2, e_3, and e_4 do not go to zero. Comparing Fig. 1 with Fig. 3, it is found that they have contrary behavior. The only reason lies in the different initial conditions. Similar result also exists by comparing Fig. 2 with Fig. 4.

Besides, we also discover the alternative CS and AS. In Fig. 5, the system shows alternative switching between these two states where the initial condition $(x_1, y_1, u_1, v_1) = (2, 5, 1, 0.3)$, $(x_2, y_2, u_2, v_2) = (-8, -9, 0, 5)$, and $p = 12$, $q = 12$.

4. Conclusions

In this paper, parameter excited chaos synchronizations of two identical double Duffing systems are studied by adjusting the strength of the substituting variable. Numerical simulations are illustrated for CS or AS of which the occurrence depends on initial conditions and driving strength. Besides, alternative CS and AS is also discovered with the same initial conditions and the same driving strength.

Acknowledgments

This research was supported by the National Science Council, Republic of China, under Grant no. NSC 96-2221-E-009-145-MY3.

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